Synchronisation Signals

[1] Analysis of the Frequency Offset Effect on Zadoff-Chu Sequence Timing Performance

[2] Zadoff-Chu Coded Ultrasonic Signal for Accurate Range Estimation

So far we have looked at generic sinusoidal burst waves for transmission across the network. When these signals are cross-correlated with an expected waveform we can see a peak at the lag position where the two signals overlap, giving us a measure for the delay time. What we can also see is a wide sideband of peaks which get larger the closer to the true peak. If we were to take the measurement for the lag be the maximum value of the waveform, we can see how a noisy signal could lead to incorrectly choosing the maximum.

There are signals with autocorrelation properties which mitigate against this. One of the properties of Additive white Gaussian noise (AWGN) is that the auto-correlation values for any non-zero delay, i.e. where the signals do not align perfectly, is effectively zero. It also has zero cross-correlation with any other AWGN waveform. This is a very useful property in peak detection. Pseudo-random noise (PRN) sequences also display similar autocorrelation properties. At zero time-delay there is a peak, and at non-zero time-delays the values are very small. These PRN signals also carry timing information as well, which is extremely useful in distributed systems to coordinating device transmissions.

In wireless transmission, a PRN sequence is used in setting up connections to detect and synchronise devices. A wireless access point will transmit a synchronisation signal. A matched filter in devices check incoming frames for this PRN sequence and aligns its local clock to the access point’s and sends back a signal, which the access point then scans for the PRN sequence, detecting the devices timing and instructs the device to adjust its transmit timing to account for round trip propagation.

So far, we have determined that a sequence with good autocorrelation properties are useful in time-of-arrival detection. However, another useful property of these sequences is having zero or very low cross-correlation with the same signal at any delay. A complex PRN sequence has a periodic autocorrelation of where N is the period of the PN sequence. Therefore, cyclically shifted PN sequences have a correlation with the original sequence.

Kasami Code - Kasami sequences are binary sequences of length 2N-1 where N is an even integer. Kasami sequences have good cross-correlation values approaching the Welch lower bound. There are two classes of Kasami sequences—the small set and the large set.

Gold Code - A Gold code, also known as Gold sequence, is a type of binary sequence, used in telecommunication (CDMA)[1] and satellite navigation (GPS).[2] Gold codes are named after Robert Gold.[3][4] Gold codes have bounded small cross-correlations within a set, which is useful when multiple devices are broadcasting in the same frequency range. A set of Gold code sequences consists of 2n − 1 sequences each one with a period of 2n − 1. Gold codes are used in GPS. The GPS C/A ranging codes are Gold code of period 1,023.

A Zadoff-Chu Sequence is a complex-valued sequence with some very useful properties in signal transmission. It is given by the equation

Where is the length of the sequence.

When is odd, the sequence is periodic  
2. If is prime, the Discrete Fourier Transform of a Zadoff–Chu sequence is another Zadoff–Chu sequence conjugated, scaled and time scaled.

{\displaystyle X\_{u}[k]=x\_{u}^{\*}({\tilde {u}}k)X\_{u}[0]} X\_{{u}}[k]=x\_{{u}}^{{\*}}({\tilde {u}}k)X\_{{u}}[0] where {\displaystyle {\tilde {u}}} {\tilde {u}} is the multiplicative inverse of u modulo {\displaystyle N\_{\text{ZC}}} N\_{{\text{ZC}}}.

3. The auto correlation of a Zadoff–Chu sequence with a cyclically shifted version of itself is zero, i.e., it is non-zero only at one instant which corresponds to the cyclic shift.

4. The cross-correlation between two prime length Zadoff–Chu sequences, i.e. different values of {\displaystyle u,u=u\_{1},u=u\_{2}} u,u=u\_{1},u=u\_{2}, is constant {\displaystyle {\sqrt {N\_{\text{ZC}}}}} {\sqrt {N\_{{\text{ZC}}}}}, provided that {\displaystyle u\_{1}-u\_{2}} u\_{1}-u\_{2} is relatively prime to . [2]

Zadoff-Chu is used in 3gPP LTE services for both synchronisation and random access preambles   
Zadoff–Chu sequences are an improvement over the Walsh–Hadamard codes used in UMTS because they result in a constant-amplitude output signal, reducing the cost and complexity of the radio's power amplifier.[3]

Supposing we had a single nodes whose location is unknown is communicating with multiple fixed point nodes whose locations are known.

The orthogonal nature of the ZC signals means that multiple cyclically shifted signals can be combined and sent simultaneously in a single transmission. If each receiver had a matched filter to look for the signal with a particular phase shift, the other signals in the transmission would not be detected. This can also

[2]

*Zadoff-Chu sequence is a polyphase complex valued sequence, named after Solomon A. Zadoff and D.C. Chu [15],[16]. Zadoff-Chu sequences are constant amplitude zero auto-correlation (CAZAC) sequences, where the autocorrelationiszeroforanynonzerolag.AZadoff-Chusequence sk of length N can be written in the following form [16]: sk = ei↵k,k =0 ,1,2,...,N1, (1) where ↵k is given by: ↵k =(M⇡k2 N , if N is even M⇡k(k+1) N , if N is odd (2) where M and N are integers and M is coprime to N. The received signal is modeled as: y[k]=↵x[kd]+w[k], (3) where ↵ is an attenuation factor, x[k] is the transmitted signal, d is the delay between transmitting and receiving the signal*

*and w[k] is an additive white Gaussian noise (AWGN). If the transmitted signal is an odd-length Zadoff-Chu sequence, y[k] can be written as: y[k]=↵ei⇡ M N (kd)(k1d) + w[k],k=1 ,2,...,N (4) The cross-correlation of x with y is given by:*

*(x⇤y)[n] ,*

*N1 X m=0*

*x⇤[m]y[m + n] (5)*

*The cross correlation function r[n] is given by:*

*r[n]= N1 X k=0*

*x⇤[k]y[k + n]*

*=*

*N1 X k=0*

*ei⇡ M N k(k+1)⇣↵ei⇡ M N (k+nd)(k+1+nd) + w[k + n]⌘ Considering the noiseless case and with some manipulations we can reduce the summation to:*

*r[n]=↵ei⇡ M N (nd)(n+1d)*

*N1 X k=0*

*ei⇡ M N 2k(nd) (6)*

*Taking the absolute value of the cross-correlation function, we have: |r[n]|=|↵| N1 X k=0 ei⇡ M N 2k(nd) = |↵| ei2⇡M(nd) 1 ei2⇡ M N (nd) 1 = (0 if n 6= d |↵|N if n = d note that M and N are coprime. The peak of the cross correlation is at the TOF (n = d). The same derivations can be implemented to even-length Zadoff-Chu sequences. Since the signal bandwidth BW = 2 Tsym, where Tsym is symbol duration, the minimum symbol duration depends on the available bandwidth. The hardware used in this work has a total bandwidth of 7 kHz, limiting the symbol duration not to be less than 0.286 ms. The sequence in this work has been chosen to have a length of 15 symbols (N = 15) and a symbol duration of 0.3125 ms, resulting a sequence duration of 4.688 ms. For continuous transmission, this sequence is repeated P times, where each single sequence is referred to as a block.*

*Assuming multi-paths reception, the received signal in (3) can be written as*

*y[k]=*

*L X i=1*

*↵ix[kdi]+w[k]. (7) A cross-correlation between a reference block and a window of the received signal is applied to estimate the TOF of the ﬁrst block. The peak associated with the direct path is not necessarily the highest peak, multi-paths can add up constructively and result in a higher peak. However, the direct path peak is always the ﬁrst signiﬁcant peak to arrive. Cross correlating the reference transmitted block with the received blocks results in peaks at d1, d2, ..., dL, with d1 <d i, i =2 ,3,...,L. In order to ﬁnd the peak corresponding to the TOF of the ﬁrst block, an early peak search is applied where the highest peak is located, then a search for all the earliest peak with amplitude greater than a threshold of 0.6 of the highest peak is performed. This threshold was tested experimentally and proved to be high enough to avoid noise, and low enough for direct path detection. Once the TOF, d1, of the direct path has been estimated, the distance to the target is given by D = d1 vs fs , (8) where vs is the speed of sound in air. Figure 2 illustrates this step.*